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METHOD FOR CALCULATING WING CHARACTERISTICS BY LIFTING-LINE THEORY USING NONLINEAR SECTION LIFT DATA

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SUMMARY

A method is presented for calculating wing characteristics by lifting-line theory using nonlinear section lift data. Material from various sources is combined with some original work into the single complete method described. Multhopp's systems of multipliers are employed to obtain the induced angle of attack directly from the spanwise lift distribution. Equations are developed for obtaining these multipliers for any even number of spanwise stations, and values are tabulated for 10 stations along the semispan for asymmetrical, symmetrical, and antisymmetrical lift distributions. In order to minimize the computing time and to illustrate the procedures involved, simplified computing forms containing detailed examples are given for symmetrical lift distributions. Similar forms for asymmetrical and antisymmetrical lift distributions, although not shown, can be readily constructed in the same manner as those given. The adaptation of the method for use with linear section lift data is also illustrated. This adaptation has been found to require less computing time than most existing methods.

The wing characteristics calculated from general nonlinear section lift data have been found to agree much closer with experimental data in the region of maximum lift coefficient than those calculated on the assumption of linear section lift curves. The calculations are subject to the limitations of lifting-line theory and should not be expected to give accurate results for wings of low aspect ratio and large amounts of sweep.

INTRODUCTION

The lifting-line theory is the best known and most readily applied theory for obtaining the spanwise lift distribution of a wing and the subsequent determination of the aerodynamic characteristics of the wing from two-dimensional airfoil data. The characteristics so determined are in fairly close agreement with experimental results for wings with small amounts of sweep and with moderate to high values of aspect ratio; for this reason, this theory has served as the basis for a large part of present aeronautical knowledge.

The hypothesis upon which the theory is based is that a lifting wing can be replaced by a lifting line and that the incremental vortices shed along the span trail behind the wing in straight lines in the direction of the free-stream velocity. The strength of these trailing vortices is proportional to the rate of change of the lift along the span. The trailing vortices induce a velocity normal to the direction of the free-stream velocity and to the lifting line. The effective angle of attack of each section of the wing is therefore different from the geometric angle of attack by the amount of the angle (called the induced angle of attack) whose tangent is the ratio of the value of the induced velocity at the lifting line to the value of the free-stream velocity. The effective angle of attack is thus related to the lift distribution through the induced angle of attack. In addition, the effective angle of attack is related to the section lift coefficient according to two-dimensional data for the airfoil sections incorporated in the wing. Both relationships must be simultaneously satisfied in the calculation of the lift distribution of the wing.

If the section lift curves are linear, these relationships may be expressed by a single equation which can be solved analytically. In general, however, the section lift curves are not linear, particularly at high angles of attack, and analytical solutions are not feasible. The method of calculating the spanwise lift distribution using nonlinear section lift data thus becomes one of making successive approximations of the lift distribution until one is found that simultaneously satisfies the aforementioned relationships.

Such a method has been used by Wieselsberger (reference 1) for the region of maximum lift coefficient and by Boshar (reference 2) for high-subsonic speeds. Both of these writers used Tani's system of multipliers for obtaining the induced angle of attack at five stations along the semispan of the wing (reference 3). Tani, however, considered only the case of wings with symmetrical lift distributions. Multhopp (reference 4), using a somewhat different mathematical treatment from that which Tani used, derived systems of multipliers for symmetrical, antisymmetrical, and asymmetrical lift distributions for 4, 8, and 16 stations along the semispan. Multhopp's derivation, in slightly different form and nomenclature, is presented herein and tables are given for the multipliers for 10 stations along the semispan (the usual number of stations considered in many reports in the United States).

For symmetrical distributions of wing chord and angle of attack, the multipliers for symmetrical lift distributions may be used with nonlinear or linear section lift curves. For asymmetrical distributions of angle of attack, the multipliers for asymmetrical lift distributions must be used if nonlinear section lift curves are used. If an asymmetrical distribution of angle of attack can be broken up into a symmetrical and an antisymmetrical distribution, the antisymmetrical part may be treated separately if the section lift curves can be assumed to be linear.

The purpose of the present paper is to combine the contributions of Multhopp and several other writers, together with some original work, into a single complete method of calculating the lift distributions and force and moment characteristics of wings, using nonlinear section lift data. Simplified computing forms are given for the calculation of symmetrical lift distributions and their use is illustrated by a detailed example. The adaptation of the method for use with linear section lift data is also illustrated. No forms are given for asymmetrical or antisymmetrical lift distributions inasmuch as such forms would be very similar to those given.

SYMBOLS

S	wing area
b	wing span
c	chord at any section
c_s	root chord
c_t	tip chord
$\bar{c}$	mean geometric chord (S/b)
c'	mean aerodynamic chord $\left(\frac{2}{S}\int_0^{b/2} c^2dy\right)$
A	aspect ratio (b^2/S)
x	coordinate parallel to root chord
y	coordinate perpendicular to plane of symmetry
z	coordinate perpendicular to root chord and parallel to plane of symmetry
q	free-stream dynamic pressure $\left(\frac{1}{2}\rho V^2\right)$
R	Reynolds number ($\rho Vc/\mu$ or $\rho Vc'/\mu$)
ρ	mass density
V	free-stream velocity
μ	coefficient of viscosity
C_L	wing lift coefficient (L/qS)
c_l	section lift coefficient (l/qc)
L	wing lift
l	section lift
C_D	wing drag coefficient (D/qS)
C_{D_0}	wing profile-drag coefficient
C_{D_i}	wing induced-drag coefficient
c_{d_0}	section profile-drag coefficient
c_{d_i}	section induced-drag coefficient
D	wing drag
C_m	wing pitching-moment coefficient (M/qSc')
$c_{m_{c/4}}$	section pitching-moment coefficient about section quarter-chord point
M	wing pitching moment
C_l	wing rolling-moment coefficient (L'/qSb)
L'	wing rolling moment
C_{n_i}	wing induced-yawing-moment coefficient
C_{n_0}	wing profile-yawing-moment coefficient
α	angle of attack of any section along the span referred to its chord line
α_s	angle of attack of root section referred to its chord line
α_{a_s}	angle of attack of root section referred to its zero lift line
α_i	section induced angle of attack
α_e	effective angle of attack of any section
α_0	section angle of attack for two-dimensional airfoils
α_{l_0}	angle of zero lift of any section
$\alpha_{l_{0_s}}$	angle of zero lift of root section
$\alpha_{s_{(L=0)}}$	wing angle of attack for zero lift
ϵ	geometric angle of twist of any section along the span (negative if washout)
ϵ'	aerodynamic angle of twist of any section along the span (negative if washout)
ϵ_t	geometric angle of twist of tip section
ϵ_t'	aerodynamic angle of twist of tip section
a	wing lift-curve slope, per degree
a_0	section lift-curve slope, per degree $\left(\frac{\text{Two-dimensional lift-curve slope}}{\text{Edge-velocity factor}}\right)$
$\cos\theta$	coordinate ($2y/b$)
A_n	coefficients in trigonometric series
β_{mk}	multiplier for induced angle of attack (asymmetrical distributions)
λ_{mk}	multiplier for induced angle of attack (symmetrical distributions)
γ_{mk}	multiplier for induced angle of attack (antisymmetrical distributions)
η_m	multiplier for lift, drag, and pitching-moment coefficients (asymmetrical distributions)
η_{ms}	multiplier for lift, drag, and pitching-moment coefficients (symmetrical distributions)
σ_m	multiplier for rolling- and yawing-moment coefficients (asymmetrical distributions)
σ_{ma}	multiplier for rolling-moment coefficient (antisymmetrical distributions)
E	edge-velocity factor $\left(\frac{\text{Semiperimeter}}{\text{Span}}\right)$

Subscripts:

max	maximum value
$a1$	value for additional lift ($C_L=1$)
b	value for basic lift ($C_L=0$)
(α_{a_s})	value for constant value of α_{a_s}
(ϵ_t')	value for given value of ϵ_t'

THEORETICAL DEVELOPMENT OF METHOD

LIFT DISTRIBUTION

The methods of Tani (reference 3) and Multhopp (reference 4) for determining the induced angle of attack are fundamentally the same, differing only in the mathematical treatment. The method presented herein is essentially the same as that given by Multhopp. In the following derivation the spanwise lift distribution is expressed as the trigonometric series

$$\frac{c_l c}{b}=\sum A_n \sin n\theta \tag{1}$$

as in reference 5, where θ is defined by the relation $\cos\theta=\frac{2y}{b}$. It may be noted that each coefficient A_n, as used herein, is equal to four times the corresponding coefficient in reference 5. The induced angle of attack (in degrees) at a point y_1 on the lifting line is

$$\alpha_i=\frac{180}{\pi}\frac{b}{8\pi}\int_{-b/2}^{b/2}\frac{\frac{d\left(\frac{c_l c}{b}\right)}{dy}}{y_1-y}dy \tag{2}$$

This integral (in different nomenclature) was given by Prandtl in reference 6. If equation (1) is substituted into equation (2) and the variable is changed from y to θ, the induced angle of attack at the general point θ becomes, according to reference 5,

$$\alpha_i=\frac{180}{4\pi\sin\theta}\sum nA_n\sin n\theta \tag{3}$$

The problem of obtaining the induced angle of attack is thus reduced to one of determining the coefficients of the trigonometric series.

The lift distribution (equation (1)) may be approximated by a finite trigonometric series of $r-1$ terms where, for subsequent usage, r is assumed to be even. The values of $c_l c/b$ at the equally spaced points $\theta=\frac{m\pi}{r}$ in the range $0<\theta<\pi$ are expressed as

$$\left(\frac{c_l c}{b}\right)_m=\sum_{n=1}^{r-1}A_n\sin n\frac{m\pi}{r} \tag{4}$$

where $m=1, 2, 3, \ldots r-1$. Conversely, if the values of $c_l c/b$ are known at each point, the coefficients A_n of the finite series may be found by harmonic analysis as

$$A_n=\frac{2}{r}\sum_{m=1}^{r-1}\left(\frac{c_l c}{b}\right)_m\sin n\frac{m\pi}{r} \tag{5}$$

If equation (5) is substituted in equation (3), a double summation is obtained for the induced angle of attack as

$$\alpha_i(\theta)=\frac{180}{4\pi\sin\theta}\left(\sum_{n=1}^{r-1}n\sin n\theta\right)\left[\frac{2}{r}\sum_{m=1}^{r-1}\left(\frac{c_l c}{b}\right)_m\sin n\frac{m\pi}{r}\right]$$

$$=\frac{180}{4\pi r\sin\theta}\sum_{m=1}^{r-1}\left(\frac{c_l c}{b}\right)_m\sum_{n=1}^{r-1}n\left[\cos n\left(\theta-\frac{m\pi}{r}\right)-\cos n\left(\theta+\frac{m\pi}{r}\right)\right]$$

If the induced angle of attack is to be determined at the same points θ at which the load distribution is known, that is, at the points $\theta=\frac{k\pi}{r}$, then

$$\alpha_{i_k}=\frac{180}{4\pi r\sin\frac{k\pi}{r}}\sum_{m=1}^{r-1}\left(\frac{c_l c}{b}\right)_m\sum_{n=1}^{r-1}n\left[\cos n\frac{(k-m)\pi}{r}-\cos n\frac{(k+m)\pi}{r}\right]$$

$$=\sum_{m=1}^{r-1}\left(\frac{c_l c}{b}\right)_m\beta_{mk} \tag{6}$$

where

$$\beta_{mk}=\frac{180}{4\pi r\sin\frac{k\pi}{r}}\sum_{n=1}^{r-1}n\left[\cos n\frac{(k-m)\pi}{r}-\cos n\frac{(k+m)\pi}{r}\right] \tag{7}$$

It can be shown that, if $\cos\phi\neq 1$,

$$\sum_{n=1}^{r-1}n\cos n\phi=\frac{r\cos(r-1)\phi-(r-1)\cos r\phi-1}{2(1-\cos\phi)}$$

If $\phi=0$, a numerical series is obtained

$$\sum_{n=1}^{r-1}n=\frac{r(r-1)}{2}$$

By use of these relationships in equation (7) it is found that when $k\pm m$ is odd,

$$\beta_{mk}=\frac{180}{4\pi r\sin\frac{k\pi}{r}}\left[\frac{1}{1-\cos\frac{(k+m)\pi}{r}}-\frac{1}{1-\cos\frac{(k-m)\pi}{r}}\right] \tag{8a}$$

when $k=m$,

$$\beta_{mk}=\frac{180r}{8\pi\sin\frac{k\pi}{r}} \tag{8b}$$

and when $k\pm m$ is even and $k\neq m$,

$$\beta_{mk}=0 \tag{8c}$$

For a symmetrical lift distribution

$$\left(\frac{c_l c}{b}\right)_m=\left(\frac{c_l c}{b}\right)_{r-m}$$

and

$$\alpha_{i_k}=\alpha_{i_{r-k}}$$

so that the summation for α_{i_k} needs to be made only from 1 to $r/2$

$$\alpha_{i_k}=\sum_{m=1}^{r/2}\left(\frac{c_l c}{b}\right)_m\lambda_{mk} \tag{9}$$

where, when $k\pm m$ is odd,

$$\lambda_{mk}=\beta_{mk}+\beta_{r-m,\,k}\quad\left(\text{for } m\neq\frac{r}{2}\right)$$

$$=\frac{180}{2\pi r\sin\frac{k\pi}{r}}\left[\frac{\cot\frac{(k+m)\pi}{r}}{\sin\frac{(k+m)\pi}{r}}-\frac{\cot\frac{(k-m)\pi}{r}}{\sin\frac{(k-m)\pi}{r}}\right] \tag{10a}$$

$$\lambda_{mk}=\beta_{mk}\quad\left(\text{for } m=\frac{r}{2}\right)$$

$$=-\frac{180}{\pi r\left(\cos\frac{2k\pi}{r}+1\right)} \tag{10b}$$

when $k=m$,

$$\lambda_{mk}=\beta_{mk}$$

$$=\frac{180r}{8\pi\sin\frac{k\pi}{r}} \tag{10c}$$

and when $k\pm m$ is even and $k\neq m$,

$$\lambda_{mk}=0 \tag{10d}$$

For an antisymmetrical lift distribution

$$\left(\frac{c_l c}{b}\right)_m = -\left(\frac{c_l c}{b}\right)_{r-m}$$

and

$$\alpha_{i_k} = -\alpha_{i_{r-k}}$$

In this case the summation for α_{i_k} needs to be made only from 1 to $\frac{r}{2}-1$ since $\left(\frac{c_l c}{b}\right)_{r/2}=0$; then

$$\alpha_{i_k} = \sum_{m=1}^{\frac{r}{2}-1} \left(\frac{c_l c}{b}\right)_m \gamma_{mk} \tag{11}$$

where, when $k \pm m$ is odd,

$$\gamma_{mk} = \beta_{mk} - \beta_{r-m,k}$$

$$= \frac{180}{2\pi r}\left[\frac{1}{\sin^2 \frac{(k+m)\pi}{r}} - \frac{1}{\sin^2 \frac{(k-m)\pi}{r}}\right] \tag{12a}$$

when $k=m$,

$$\gamma_{mk} = \beta_{mk}$$

$$= \frac{180r}{8\pi \sin \frac{k\pi}{r}} \tag{12b}$$

and when $k \pm m$ is even and $k \neq m$,

$$\gamma_{mk} = 0 \tag{12c}$$

Multipliers can thus be calculated so that the induced angle may be readily obtained by multiplying the known values of $c_l c/b$ by the appropriate multipliers and adding the resulting products. The multipliers are independent of the aspect ratio and taper ratio of the wing. Tables I and II present values of β_{mk}, and λ_{mk} and γ_{mk}, respectively, for $r=20$. Similar tables for $\frac{4\pi}{180}\lambda_{mk}$ and $\frac{4\pi}{180}\gamma_{mk}$ are given in

TABLE I.—INDUCED-ANGLE-OF-ATTACK MULTIPLIERS β_{mk} FOR ASYMMETRICAL LIFT DISTRIBUTIONS[1]

$$\left[\alpha_{i_k} = \sum_{m=1}^{19} \left(\frac{c_l c}{b}\right)_m \beta_{mk}\right]$$

	$\frac{2y}{b}$	−0.9877	−0.9511	−0.8910	−0.8090	−0.7071	−0.5878	−0.4540	−0.3090	−0.1564	0		
$\frac{2y}{b}$	m \ k	19	18	17	16	15	14	13	12	11	10		
−0.9877	19	915.651	−166.985	0	−7.019	0	−1.401	0	−0.486	0	−0.230	1	0.9877
−.9511	18	−329.859	463.533	−122.749	0	−7.438	0	−1.792	0	−.701	0	2	.9511
−.8910	17	0	−180.336	315.512	−96.737	0	−7.073	0	−1.920	0	−.819	3	.8910
−.8090	16	−26.374	0	−125.246	243.694	−81.067	0	−6.680	0	−1.977	0	4	.8090
−.7071	15	0	−17.020	0	−97.524	202.571	−71.139	0	−6.391	0	−2.026	5	.7071
−.5878	14	−7.246	0	−12.604	0	−81.392	177.054	−64.735	0	−6.228	0	6	.5878
−.4540	13	0	−5.166	0	−10.126	0	−71.296	160.761	−60.725	0	−6.192	7	.4510
−.3090	12	−2.958	0	−4.022	0	−8.596	0	−64.817	150.611	−58.514	0	8	.3090
−.1564	11	0	−2.241	0	−3.322	0	−7.604	0	−60.708	145.025	−57.812	9	.1564
0	10	−1.468	0	−1.804	0	−2.865	0	−6.950	0	−58.533	143.239	10	0
.1564	9	0	−1.153	0	−1.518	0	−2.554	0	−6.530	0	−57.812	11	−.1564
.3090	8	−.810	0	−.946	0	−1.319	0	−2.340	0	−6.288	0	12	−.3090
.4540	7	0	−.646	0	−.800	0	−1.176	0	−2.192	0	−6.192	13	−.4540
.5878	6	−.467	0	−.530	0	−.691	0	−1.068	0	−2.092	0	14	−.5878
.7071	5	0	−.368	0	−.441	0	−.604	0	−.981	0	−2.026	15	−.7071
.8090	4	−.261	0	−.291	0	−.366	0	−.528	0	−.903	0	16	−.8090
.8910	3	0	−.192	0	−.225	0	−.297	0	−.452	0	−.819	17	−.8910
.9511	2	−.118	0	−.130	0	−.161	0	−.224	0	−.361	0	18	−.9511
.9877	1	0	−.060	0	−.069	0	−.090	0	−.133	0	−.230	19	−.9877
		1	2	3	4	5	6	7	8	9	10	k \ m	$\frac{2y}{b}$
		.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564	0	$\frac{2y}{b}$	

[1] Values of k at top to be used with values of m at left side; values of k at bottom to be used with values of m at right side.

TABLE II.—INDUCED-ANGLE-OF-ATTACK MULTIPLIERS λ_{mk} FOR SYMMETRICAL LIFT DISTRIBUTIONS AND γ_{mk} FOR ANTISYMMETRICAL LIFT DISTRIBUTIONS

$\frac{2y}{b}$	$\frac{2y}{b}$ →	0	0.1564	0.3090	0.4540	0.5878	0.7071	0.8090	0.8910	0.9511	0.9877
	m \ k	10	9	8	7	6	5	4	3	2	1
		Multipliers λ_{mk}					$\alpha_{i_k}=\sum_{m=1}^{10}\left(\frac{c_lc}{b}\right)_m\lambda_{mk}$				
0	10	143.239	−58.533	0	−6.950	0	−2.865	0	−1.804	0	−1.468
.1564	9	−115.624	145.025	−67.298	0	−10.158	0	−4.840	0	−3.394	0
.3090	8	0	−64.302	150.611	−67.157	0	−9.916	0	−4.968	0	−3.768
.4540	7	−12.384	0	−62.917	160.761	−72.472	0	−10.926	0	−5.812	0
.5878	6	0	−8.320	0	−65.803	177.054	−82.083	0	−13.134	0	−7.713
.7071	5	−4.051	0	−7.372	0	−71.743	202.571	−97.965	0	−17.388	0
.8090	4	0	−2.880	0	−7.208	0	−81.434	243.694	−125.537	0	−26.635
.8910	3	−1.638	0	−2.371	0	−7.370	0	−96.362	315.512	−180.528	0
.9511	2	0	−1.062	0	−2.016	0	−7.599	0	−122.880	463.533	−329.976
.9877	1	−.459	0	−.620	0	−1.491	0	−7.089	0	−167.045	915.651
		Multipliers γ_{mk}					$\alpha_{i_k}=\sum_{m=1}^{9}\left(\frac{c_lc}{b}\right)_m\gamma_{mk}$				
0.1564	9		145.025	−54.237	0	−5.049	0	−1.804	0	−1.087	0
.3090	8		−52.226	150.611	−62.477	0	−7.277	0	−3.076	0	−2.147
.4540	7		0	−58.533	160.761	−70.120	0	−9.326	0	−4.519	0
.5878	6		−4.136	0	−63.668	177.054	−80.701	0	−12.074	0	−6.779
.7071	5		0	−5.410	0	−70.535	202.571	−97.084	0	−16.651	0
.8090	4		−1.074	0	−6.152	0	−80.701	243.694	−124.955	0	−26.113
.8910	3		0	−1.468	0	−6.775	0	−96.512	315.512	−180.145	0
.9511	2		−.340	0	−1.567	0	−7.277	0	−122.619	463.533	−329.741
.9877	1		0	−.353	0	−1.311	0	−6.950	0	−166.926	915.651

references 7 and 8, respectively, but no derivation is given therein. Tables for $\frac{2\pi}{180}\beta_{mk}$, $\frac{2\pi}{180}\lambda_{mk}$, and $\frac{2\pi}{180}\gamma_{mk}$ are given in reference 4 for values of $r=8$, 16, and 32. An inspection of tables I and II shows that positive values occur only on the diagonal from upper left to lower right and that almost half of the values are equal to zero. The multipliers β_{mk} and λ_{mk} may be used with either nonlinear or linear section lift data, whereas the multipliers for γ_{mk} may be used only with linear section lift data.

The method of determining the lift distribution becomes one of successive approximations. For a given geometric angle of attack, a distribution of c_l is assumed from which the load distribution c_lc/b is obtained. The induced angle of attack is then determined by equation (6), (9), or (11) through the use of the appropriate multipliers and subtracted from the geometric angle of attack to give the effective angle of attack at each spanwise station. From section data for the appropriate airfoil section and local Reynolds number, values of c_l are read which correspond to the effective angle of attack of each section. If these values of c_l do not agree with those originally assumed, a second assumption is made for c_l and the process is repeated. Further assumptions are made until the assumed values of c_l are in agreement with those obtained from the section data.

WING CHARACTERISTICS

Once the lift distribution of a wing has been determined, the main part of the problem of calculating the wing characteristics is completed. The induced-drag and induced-yawing-moment coefficients are entirely dependent upon the lift distribution and it is assumed that the section profile-drag and pitching-moment coefficients are the same functions of the lift coefficient at each section of the wing as those determined in two-dimensional tests.

The calculation of each of the wing coefficients involves a spanwise integration of the distribution of a particular function $f\left(\frac{2y}{b}\right)$. This integration can be performed numerically through the use of additional sets of multipliers which are found in the following manner.

If

$$f\left(\frac{2y}{b}\right)=f(\cos\theta)=\Sigma A_n \sin n\theta$$

then

$$\int_{-1}^{1} f\left(\frac{2y}{b}\right) d\left(\frac{2y}{b}\right)=\int_0^{\pi} (\Sigma A_n \sin n\theta) \sin\theta\, d\theta$$

$$=\frac{\pi}{2} A_1$$

Since the values of $f\left(\frac{2y}{b}\right)$ are determined at the points $\theta=\frac{m\pi}{r}$, A_1 can be found by harmonic analysis as in equation (5)

$$A_1=\frac{2}{r}\sum_{m=1}^{r-1} f\left(\frac{2y}{b}\right)_m \sin\frac{m\pi}{r}$$

Therefore

$$\int_{-1}^{1} f\left(\frac{2y}{b}\right) d\left(\frac{2y}{b}\right)=\frac{\pi}{r}\sum_{m=1}^{r-1} f\left(\frac{2y}{b}\right)_m \sin\frac{m\pi}{r}$$

$$=2\sum_{m=1}^{r-1} f\left(\frac{2y}{b}\right)_m \eta_m \tag{13a}$$

where

$$\eta_m=\frac{\pi}{2r}\sin\frac{m\pi}{r}$$

If the distribution is symmetrical, $f\left(\frac{2y}{b}\right)_m=f\left(\frac{2y}{b}\right)_{r-m}$ and

$$\int_{-1}^{1} f\left(\frac{2y}{b}\right) d\left(\frac{2y}{b}\right)=2\sum_{m=1}^{r/2} f\left(\frac{2y}{b}\right)_m \eta_{ms} \tag{13b}$$

where

$$\eta_{ms}=2\eta_m \quad \left(m\neq\frac{r}{2}\right)$$

$$\eta_{ms}=\eta_m \quad \left(m=\frac{r}{2}\right)$$

The moment of the distribution $f\left(\frac{2y}{b}\right)$ can be found in a similar manner

$$\int_{-1}^{1} f\left(\frac{2y}{b}\right)\left(\frac{2y}{b}\right) d\left(\frac{2y}{b}\right)=\int_0^{\pi} (\Sigma A_n \sin n\theta) \sin\theta \cos\theta\, d\theta$$

$$=\frac{\pi}{4} A_2$$

$$=\frac{\pi}{2r}\sum_{m=1}^{r-1} f\left(\frac{2y}{b}\right)_m \sin\frac{2m\pi}{r}$$

$$=4\sum_{m=1}^{r-1} f\left(\frac{2y}{b}\right)_m \sigma_m \tag{14a}$$

where

$$\sigma_m=\frac{\pi}{8r}\sin\frac{2m\pi}{r}$$

If the distribution is antisymmetrical, $f\left(\frac{2y}{b}\right)_m=-f\left(\frac{2y}{b}\right)_{r-m}$,

$$\int_{-1}^{1} f\left(\frac{2y}{b}\right)\left(\frac{2y}{b}\right) d\left(\frac{2y}{b}\right)=4\sum_{m=1}^{\frac{r}{2}-1} f\left(\frac{2y}{b}\right)_m \sigma_{ma} \tag{14b}$$

where

$$\sigma_{ma}=2\sigma_m$$

Values of η_m, η_{ms}, σ_m, and σ_{ma} are given in table III for $r=20$.

TABLE III.—WING-COEFFICIENT MULTIPLIERS

$\frac{2y}{b}$	m	η_m	η_{ms}	σ_m	σ_{ma}
−0.9877	19	0.01229		−0.00607	
−.9511	18	.02427		−.01154	
−.8910	17	.03066		−.01589	
−.8090	16	.04616		−.01867	
−.7071	15	.05554		−.01964	
−.5878	14	.06354		−.01867	
−.4540	13	.06998		−.01589	
−.3090	12	.07470		−.01154	
−.1564	11	.07757		−.00607	
0	10	.07854	0.07854	0	0
.1564	9	.07757	.15515	.00607	.01214
.3090	8	.07470	.14939	.01154	.02308
.4540	7	.06998	.13996	.01589	.03177
.5878	6	.06354	.12708	.01867	.03735
.7071	5	.05554	.11107	.01964	.03927
.8090	4	.04616	.09233	.01867	.03735
.8910	3	.03066	.07131	.01589	.03177
.9511	2	.02427	.04854	.01154	.02308
.9877	1	.01229	.02457	.00607	.01214

Wing lift coefficient.—The wing lift coefficient is obtained by means of a spanwise integration of the lift distribution,

$$C_L=\frac{1}{S}\int_{-b/2}^{b/2} c_l c\, dy=\frac{A}{2}\int_{-1}^{1}\frac{c_l c}{b}\, d\left(\frac{2y}{b}\right)$$

For asymmetrical lift distributions

$$C_L=A\sum_{m=1}^{r-1}\left(\frac{c_l c}{b}\right)_m \eta_m \tag{15a}$$

For symmetrical lift distributions

$$C_L=A\sum_{m=1}^{r/2}\left(\frac{c_l c}{b}\right)_m \eta_{ms} \tag{15b}$$

Induced-drag coefficient.—The section induced-drag coefficient is equal to the product of the section lift coefficient and the induced angle of attack in radians,

$$c_{d_i}=\frac{\pi c_l \alpha_i}{180}$$

The wing induced-drag coefficient is obtained by means of a spanwise integration of the section induced-drag coefficient multiplied by the local chord,

$$C_{D_i}=\frac{1}{S}\int_{-b/2}^{b/2}\frac{\pi c_l c \alpha_i}{180}\, dy$$

$$=\frac{A}{2}\int_{-1}^{1}\frac{c_l c}{b}\frac{\pi\alpha_i}{180}\, d\left(\frac{2y}{b}\right)$$

For asymmetrical lift distributions

$$C_{D_i}=\frac{\pi A}{180}\sum_{m=1}^{r-1}\left(\frac{c_l c}{b}\alpha_i\right)_m \eta_m \tag{16a}$$

For symmetrical lift distributions

$$C_{D_i}=\frac{\pi A}{180}\sum_{m=1}^{r/2}\left(\frac{c_l c}{b}\alpha_i\right)_m \eta_{ms} \tag{16b}$$

Profile-drag coefficient.—The section profile-drag coefficient can be obtained from section data for the appropriate airfoil section and local Reynolds number. For each spanwise station the profile-drag coefficient is read at the section lift coefficient previously determined. The wing profile-drag coefficient is then obtained by means of a spanwise integration of the section profile-drag coefficient multiplied by the local chord,

$$C_{D_0}=\frac{1}{S}\int_{-b/2}^{b/2} c_{d_0} c\, dy$$
$$=\frac{1}{2}\int_{-1}^{1} c_{d_0}\frac{c}{\bar{c}}\, d\left(\frac{2y}{b}\right)$$

For asymmetrical lift distributions

$$C_{D_0}=\sum_{m=1}^{r-1}\left(c_{d_0}\frac{c}{\bar{c}}\right)_m \eta_m \tag{17a}$$

For symmetrical lift distributions

$$C_{D_0}=\sum_{m=1}^{r/2}\left(c_{d_0}\frac{c}{\bar{c}}\right)_m \eta_{ms} \tag{17b}$$

Pitching-moment coefficient.—The section pitching-moment coefficient about its quarter-chord point can be obtained from section data for the appropriate airfoil section and local Reynolds number. For each spanwise station the pitching-moment coefficient is read at the section lift coefficient previously determined and then transferred to the wing reference point by the equation

$$c_m=c_{m_{c/4}}-\frac{x}{c}\left[c_l\cos(\alpha_s-\alpha_i)+c_{d_0}\sin(\alpha_s-\alpha_i)\right]$$
$$-\frac{z}{c}\left[c_l\sin(\alpha_s-\alpha_i)-c_{d_0}\cos(\alpha_s-\alpha_i)\right] \tag{18}$$

where x and z are measured from the wing reference point to the quarter-chord point of the section under consideration, and upward and backward forces and distances are taken as positive. The section pitching-moment coefficient about its aerodynamic center may be used instead of $c_{m_{c/4}}$, in which case x and z are measured to the section aerodynamic center. The term $c_{d_0}\sin(\alpha_s-\alpha_i)$ may usually be neglected. The wing pitching-moment coefficient is obtained by the spanwise integration

$$C_m=\frac{1}{Sc'}\int_{-b/2}^{b/2} c_m c^2 dy$$
$$=\frac{1}{2}\int_{-1}^{1}\left(\frac{c_m c^2}{\bar{c}c'}\right)d\left(\frac{2y}{b}\right)$$

For asymmetrical lift distributions

$$C_m=\sum_{m=1}^{r-1}\left(\frac{c_m c^2}{\bar{c}c'}\right)_m \eta_m \tag{19a}$$

For symmetrical lift distributions

$$C_m=\sum_{m=1}^{r/2}\left(\frac{c_m c^2}{\bar{c}c'}\right)_m \eta_{ms} \tag{19b}$$

Rolling-moment coefficient.—The rolling-moment coefficient is obtained by means of a spanwise integration

$$C_l=-\frac{1}{Sb}\int_{-b/2}^{b/2} c_l c y\, dy$$
$$=-\frac{A}{4}\int_{-1}^{1}\frac{c_l c}{b}\frac{2y}{b}\, d\left(\frac{2y}{b}\right)$$
$$=-A\sum_{m=1}^{r-1}\left(\frac{c_l c}{b}\right)_m \sigma_m \tag{20a}$$

For an antisymmetrical lift distribution

$$C_l=-A\sum_{m=1}^{\frac{r}{2}-1}\left(\frac{c_l c}{b}\right)_m \sigma_{ma} \tag{20b}$$

Induced-yawing-moment coefficient.—The induced-yawing-moment coefficient is due to the moment of the induced-drag distribution,

$$C_{n_i}=\frac{1}{Sb}\int_{-b/2}^{b/2}\frac{\pi c_l c\alpha_i}{180}y\, dy$$
$$=\frac{A}{4}\int_{-1}^{1}\frac{c_l c}{b}\frac{\pi\alpha_i}{180}\frac{2y}{b}\, d\left(\frac{2y}{b}\right)$$
$$=\frac{\pi A}{180}\sum_{m=1}^{r-1}\left(\frac{c_l c}{b}\alpha_i\right)_m \sigma_m \tag{21}$$

The induced-yawing-moment coefficient for an antisymmetrical lift distribution is equal to zero and has little meaning inasmuch as the lift coefficient is also zero. The induced-yawing-moment coefficient is a function of the lift and rolling-moment coefficients and must be found for asymmetrical lift distributions.

Profile-yawing-moment coefficient.—The profile-yawing-moment coefficient is due to the moment of the profile-drag distribution,

$$C_{n_0}=\frac{1}{Sb}\int_{-b/2}^{b/2} c_{d_0} c y\, dy$$
$$=\frac{1}{4}\int_{-1}^{1}\frac{c_{d_0}c}{\bar{c}}\frac{2y}{b}\, d\left(\frac{2y}{b}\right)$$
$$=\sum_{m=1}^{r-1}\left(\frac{c_{d_0}c}{\bar{c}}\right)_m \sigma_m \tag{22}$$

APPLICATION OF METHOD USING NONLINEAR SECTION LIFT DATA FOR SYMMETRICAL LIFT DISTRIBUTIONS

The method described is applied herein to a wing, the geometric characteristics of which are given in table IV. Only symmetrical lift distributions are considered hereinafter inasmuch as these are believed to be sufficient for illustrating

TABLE IV.—GEOMETRIC CHARACTERISTICS OF EXAMPLE WING

Taper ratio, c_s/c_t	2.5	Root section	NACA 4420
Aspect ratio, A	10.05	Tip section	NACA 4412
Span, b, ft	15.00	Geometric twist, ϵ_t, deg	−3.50
Area, S, sq ft	22.39	Aerodynamic twist, ϵ_t', deg	−3.40
Root chord, c_s, ft	2.143	Edge velocity factor, E	1.044
Mean geometric chord, $\bar{c}$, ft	1.493	Wing Reynolds number, R	3.49×10^6
Mean aerodynamic chord, c', ft	1.592	$\alpha_{l_{0_s}}$, deg	−3.90

$\frac{2y}{b}$	$\frac{t}{c}$	R	$\frac{c}{c_s}$	$\frac{c}{b}$	$\frac{c}{\bar{c}}$	$\frac{c^2}{\bar{c}c'}$	a_0	$\frac{a_0c}{b}$	$\frac{\epsilon}{\epsilon_t}$	ϵ, deg	ϵ', deg
0	0.200	4.70×10^6	1.0000	0.1429	1.435	1.932	0.0969	0.01385	0	0	0
.1564	.195	4.26	.9062	.1295	1.300	1.586	.0973	.01260	.0690	−.24	−.235
.3090	.188	3.83	.8146	.1164	1.169	1.282	.0978	.01138	.1517	−.53	−.516
.4540	.180	3.42	.7276	.1040	1.044	1.022	.0984	.01023	.2496	−.87	−.849
.5878	.171	3.04	.6473	.0925	.929	.809	.0991	.00917	.3632	−1.27	−1.235
.7071	.161	2.70	.5757	.0823	.826	.640	.0999	.00822	.4913	−1.72	−1.670
.8090	.150	2.42	.5146	.0735	.739	.512	.1007	.00740	.6288	−2.20	−2.138
.8910	.139	2.18	.4654	.0665	.668	.418	.1014	.00674	.7658	−2.68	−2.604
.9511	.129	2.02	.4293	.0613	.616	.356	.1020	.00625	.8862	−3.10	−3.013
.9877	.123	1.44	.3061	.0437	.439	.181	.1021	.00446	.9698	−3.39	−3.297

For tapered wings with straight-line elements from root to construction tip:

$$\frac{c}{c_s}=1-\left(1-\frac{c_t}{c_s}\right)\frac{2y}{b} \qquad \left(\frac{\epsilon}{\epsilon_t}\right)=\frac{c_t}{c_s}\frac{2y/b}{c/c_s}$$

(Alter values of c/c_s near tip to allow for rounding.) (Use value of c/c_s before rounding tip.)

the method of calculation. The lift, profile-drag, and pitching-moment coefficients for the various wing sections along the span were derived from unpublished airfoil data obtained in the Langley two-dimensional low-turbulence pressure tunnel. The original airfoil data were cross-plotted against Reynolds number and thickness ratio inasmuch as both varied along the span of the wing. Sample curves are given in figures 1 and 2. From these plots the section characteristics at the various spanwise stations were determined and plotted in the conventional manner. (See fig. 3.) The edge-velocity factor E, derived in reference 9 for an elliptic wing, has been applied to the section angle of attack for each value of section lift coefficient as follows:

$$\alpha_e=E(\alpha_0-\alpha_{l_0})+\alpha_{l_0}$$

LIFT DISTRIBUTION

Computation of the lift distribution at an angle of attack of 3° is shown in table V. This table is designed to be used where the multiplication is done by means of a slide rule or simple calculating machine. Where calculating machines capable of performing accumulative multiplication are available, the spaces for the individual products in columns ⑥ to ⑮ may be omitted and the table made smaller. (See tables VII and VIII.) The mechanics of computing are explained in the table; however, the method for approximating the lift-coefficient distribution requires some explanation. The initially assumed lift-coefficient distribution (column ③ of first division) can be taken as the distribution given by the geometric angles of attack but it is best determined by some simple method which will give a close approximation to the actual distribution. The initial distribution given in table V was approximated by

$$c_l=\frac{A}{A+1.8}\left[\frac{1}{2}+\frac{2\bar{c}}{\pi c}\sqrt{1-\left(\frac{2y}{b}\right)^2}\right]c_{l(\alpha)}$$

where $c_{l(\alpha)}$ is the lift coefficient read from the section curves for the geometric angles of attack. This equation weights the lift distribution according to the average of the chord distribution of the wing under consideration and that of an elliptic wing of the same aspect ratio and span. When the lift distributions at several angles of attack are to be computed and after they have been obtained for two angles, the initially assumed c_l distribution for subsequent angles can be more accurately estimated in the following manner: Values of downwash angle are first estimated by extrapolating from values for the preceding wing angles, and then, for the resulting effective angles of attack, the lift coefficients are read from the section curves.

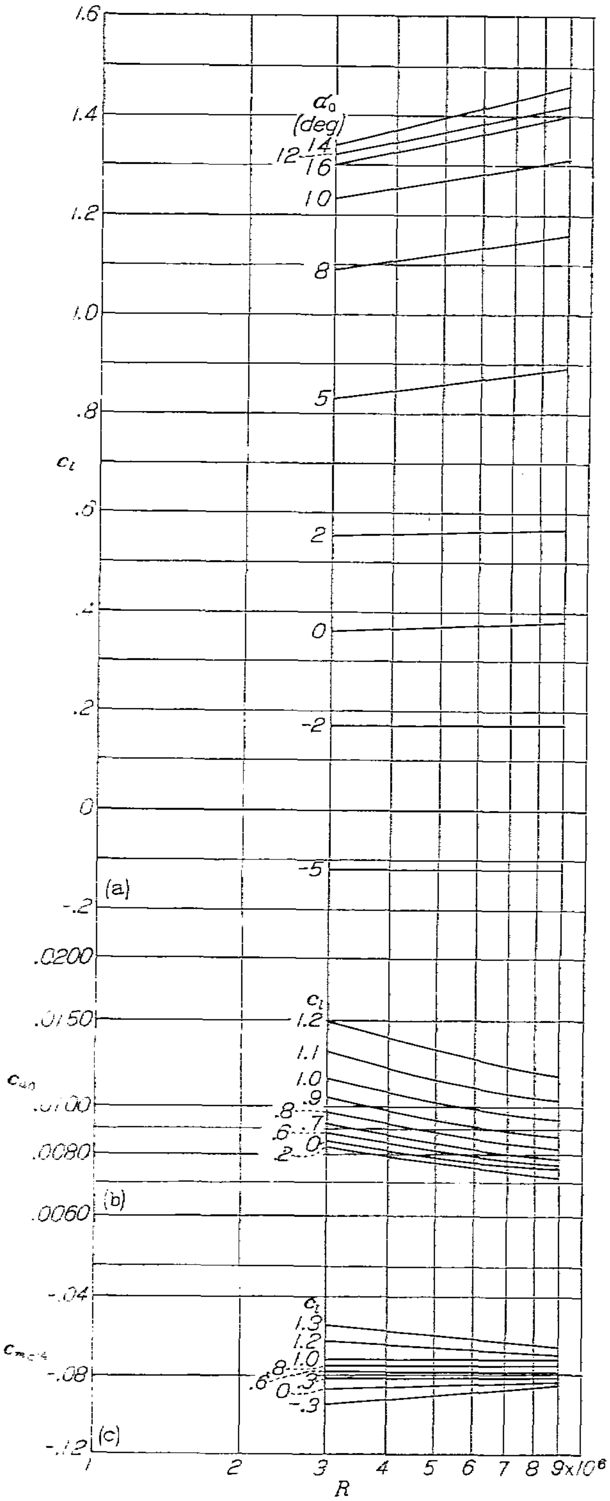
The lift coefficients in column ⑱ of table V, read from section lift curves for the effective angles of attack, will usually not check the assumed values for the first approximation. In order to select assumed values for subsequent approximations, the following simple method has been found to yield satisfactory results. An incremental value of lift coefficient Δc_{l_m} is obtained according to the following relation

$$\Delta c_{l_m}=\frac{(⑱-③)_{m-1}+3(⑱-③)_m+(⑱-③)_{m+1}}{K}$$

where circled numbers represent column numbers in table V and where K has the following values at the spanwise stations

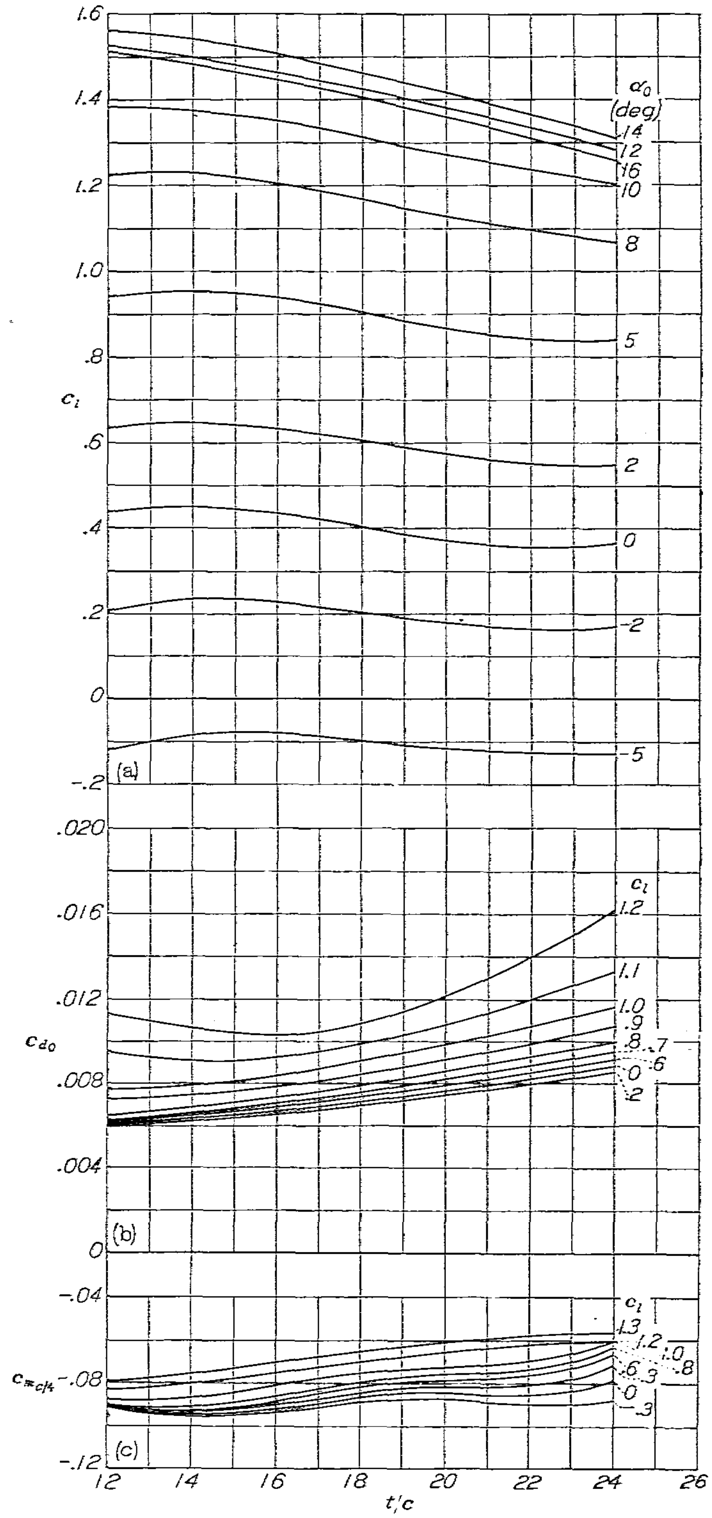
$\frac{2y}{b}$	K
0 to 0.8910	8 to 10
.9511	11 to 13
.9877	14 to 16

and $(⑱-③)_m$ is the difference between the check and assumed values for the mth spanwise station. The incremental values so determined are added to the assumed values in order to obtain new assumed values to be used in the next approximation. This method has been found in practice to make the check and assumed values converge in about



(a) Lift.
(b) Drag.
(c) Pitching moment.

FIGURE 1.—Variation of characteristics of NACA 4421 airfoil with Reynolds number. (Similar curves plotted for each thickness ratio.)



(a) Lift.
(b) Drag.
(c) Pitching moment.

FIGURE 2.—Variation of characteristics of NACA 44-series airfoil with thickness ratio. $R=4.70\times10^6$; $\frac{2y}{b}=0$. (Similar curves plotted for Reynolds numbers corresponding to each station.)

three approximations if the first approximation is not too much in error.

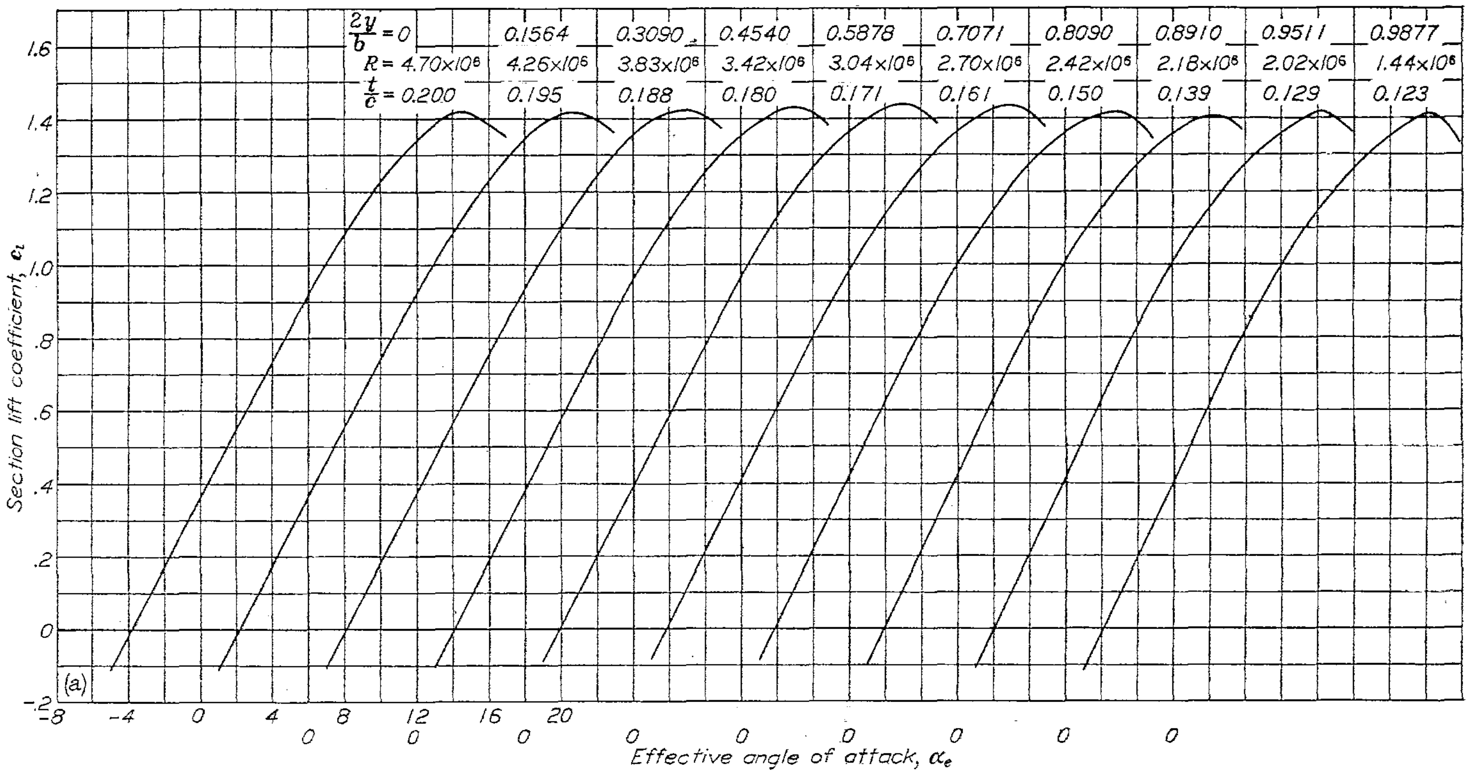
WING COEFFICIENTS

Computations of the wing lift, profile-drag, induced-drag, and pitching-moment coefficients are shown in table VI. Since the lateral axis through the wing reference point contains the quarter-chord points of each section, the x and z distances in equation (18) are zero, and the pitching-moment coefficient of the wing is determined solely by the values of $c_{m_{c/4}}$.

APPLICATION OF METHOD USING LINEAR SECTION LIFT DATA FOR SYMMETRICAL LIFT DISTRIBUTIONS

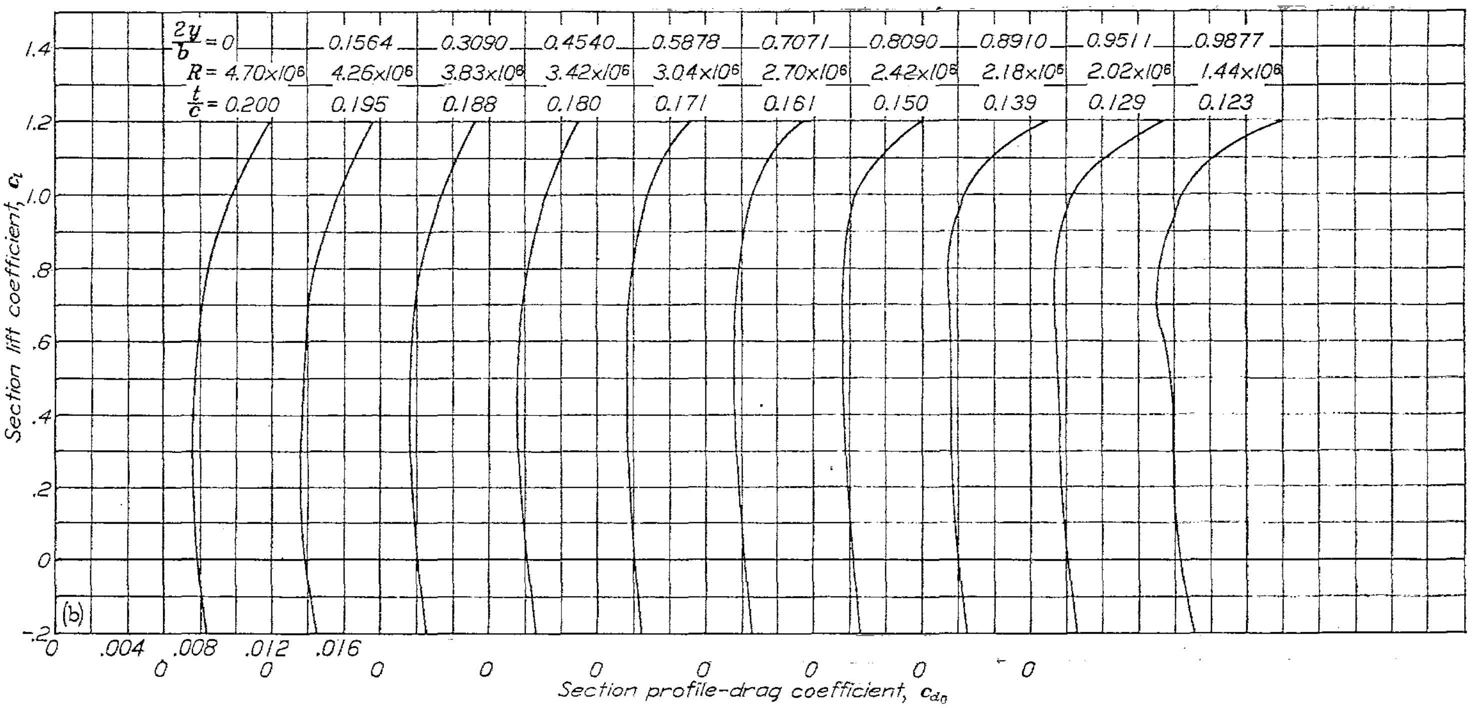
Although the method described herein was developed particularly for use with nonlinear section lift data, it is readily adaptable for use with linear section lift data with a resulting reduction in computing time as compared with most existing methods. When the section lift curves can be assumed linear, it is usually convenient to divide any symmetrical lift distribution (as in reference 10) into two parts—the additional lift distribution due to angle-of-attack changes and the basic lift distribution due to aerodynamic twist. The calculation of these lift distributions is illustrated in tables VII to X for the wing, the geometric characteristics of which were given in table IV.

It should be noted that tables VII and VIII are essentially the same as table V but are designed primarily for use with calculating machines capable of performing accumulative multiplication. If such machines are not available, these tables may be constructed similar to table V to allow spaces for writing the individual products.



(a) Lift.

FIGURE 3.—Section characteristics of example wing.



(b) Drag.

FIGURE 3.—Continued.

(c) Pitching moment.

FIGURE 3.—Concluded.

LIFT CHARACTERISTICS

Two lift distributions are required for the determination of the additional and basic lift distributions. The first one is obtained in table VII for a constant angle of attack ($\epsilon'=0$) and the second one in table VIII for the angle of attack distribution due to the aerodynamic twist ($\alpha_{a_s}=0$). The check values of c_lc/b (column ⑱) are obtained by multiplying the effective angle of attack α_e by a_0c/b. The final approximations are entered in table IX as $\left(\frac{c_lc}{b}\right)_{(\alpha_{a_s})}$ and $\left(\frac{c_lc}{b}\right)_{(\epsilon_t')}$.

The $\left(\frac{c_lc}{b}\right)_{(\alpha_{a_s})}$ distribution is the additional lift distribution corresponding to a wing lift coefficient $C_{L(\alpha_{a_s})}$ determined in table IX through the use of the multipliers η_{ms}. It is usually convenient to use the additional lift distribution $\frac{c_{l_{a1}}c}{b}$ corresponding to a wing lift coefficient of unity. This distribution is found by dividing the values of $\left(\frac{c_lc}{b}\right)_{(\alpha_{a_s})}$ by $C_{L(\alpha_{a_s})}$.

The $\left(\frac{c_lc}{b}\right)_{(\epsilon_t')}$ distribution is a combination of the basic lift distribution and an additional lift distribution corresponding to a wing lift coefficient $C_{L(\epsilon_t')}$ also determined in table IX. The basic lift distribution $\frac{c_{l_b}c}{b}$ is then determined by subtracting the additional lift distribution $\frac{c_{l_{a1}}c}{b}C_{L(\epsilon_t')}$ from $\left(\frac{c_lc}{b}\right)_{(\epsilon_t')}$.

Inasmuch as the wing lift curve is assumed to be linear, it is defined by its slope and angle of attack for zero lift which are also found in table IX. The maximum wing lift coefficient is estimated according to the method of reference 10 which is illustrated in figure 4. The maximum lift coefficient is considered to be the wing lift coefficient at which some section of the wing becomes the first to reach its maximum lift, that is, $c_{l_b}+C_L\,c_{l_{a1}}=c_{l_{max}}$. This value of C_L is most conveniently determined by finding the minimum value of $\frac{c_{l_{max}}-c_{l_b}}{c_{l_{a1}}}$ along the span as illustrated in table IX.

INDUCED-DRAG COEFFICIENT

The section induced-drag coefficient is equal to the product of the section lift coefficient and the induced angle of attack in radians. The lift distribution for any wing lift coefficient is

$$\frac{c_lc}{b}=\frac{c_{l_{a1}}c}{b}C_L+\frac{c_{l_b}c}{b} \tag{23}$$

The corresponding induced angle of attack distribution may be written as

$$\alpha_i=\alpha_{i_{a1}}C_L+\alpha_{i_b} \tag{24}$$

TABLE V.—CALCULATION OF LIFT DISTRIBUTION FOR EXAMPLE WING

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
	$\frac{2y}{b}$	α $(\alpha_i+\epsilon)$	c_l (Assumed)	$\frac{c}{b}$ (Table IV)	$\frac{c_l c}{b}$ (③×④)	$\lambda_{mk}\times$⑤										α_i (Σ⑥ to Σ⑮)	α_e (②−⑯)	Check c_l
						0	.1564	.3090	.4540	.5878	.7071	.8090	.8910	.9511	.9877			
First approximation						143.239	−58.533	0	−6.950	0	−2.865	0	−1.804	0	−1.468			
	0	3.00	0.513	0.1429	0.0733	10.50	−4.29	0	−.51	0	−.21	0	−.13	0	−.11	1.88	1.12	0.464
						−115.624	145.025	−67.298	0	−10.158	0	−4.840	0	−3.394	0			
	.1564	2.76	.517	.1295	.0670	−7.75	9.72	−4.51	0	−.68	0	−.32	0	−.23	0	.98	1.78	.531
						0	−64.802	150.611	−67.157	0	−9.916	0	−4.968	0	−3.768			
	.3090	2.47	.523	.1164	.0609	0	−3.95	9.17	−4.09	0	−.60	0	−.30	0	−.23	.90	1.57	.522
						−12.384	0	−62.917	160.761	−72.472	0	−10.926	0	−5.812	0			
	.4540	2.13	.519	.1040	.0540	−.67	0	−3.40	8.68	−3.91	0	−.59	0	−.31	0	.77	1.36	.514
						0	−8.320	0	−65.803	177.054	−82.083	0	−13.134	0	−7.713			
	.5878	1.73	.501	.0925	.0463	0	−.39	0	−3.05	8.20	−3.80	0	−.61	0	−.36	.60	1.13	.500
						−4.051	0	−7.372	0	−71.743	202.571	−97.965	0	−17.388	0			
	.7071	1.28	.477	.0823	.0393	−.16	0	−.29	0	−2.82	7.96	−3.85	0	−.68	0	.65	.63	.474
						0	−2.880	0	−7.208	0	−81.434	243.694	−125.537	0	−26.635			
	.8090	.80	.430	.0735	.0316	0	−.09	0	−.23	0	−2.57	7.70	−3.97	0	−.84	.55	.25	.449
						−1.638	0	−2.371	0	−7.370	0	−96.962	315.512	−180.528	0			
	.8910	.32	.360	.0665	.0239	−.04	0	−.06	0	−.18	0	−2.32	7.54	−4.31	0	.42	−.10	.413
						0	−1.062	0	−2.016	0	−7.599	0	−122.880	463.533	−329.976			
	.9511	−.10	.281	.0613	.0172	0	−.02	0	−.03	0	−.13	0	−2.11	7.97	−5.68	.77	−.87	.326
						−.459	0	−.620	0	−1.491	0	−7.089	0	−167.045	915.651			
	.9877	−.39	.228	.0437	.0100	0	0	−.01	0	−.01	0	−.07	0	−1.67	9.16	1.94	−2.33	.165
					Σ	1.88	.98	.90	.77	.60	.65	.55	.42	.77	1.94			
Second approximation						143.239	−58.533	0	−6.950	0	−2.865	0	−1.804	0	−1.468			
	0	3.00	0.498	0.1429	0.0712	10.20	−4.17	0	−.49	0	−.20	0	−.13	0	−.10	1.61	1.39	.491
						−115.624	145.025	−67.298	0	−10.158	0	−4.840	0	−3.394	0			
	.1564	2.76	.516	.1295	.0668	−7.72	9.69	−4.50	0	−.68	0	−.32	0	−.23	0	1.07	1.69	.523
						0	−64.802	150.611	−67.157	0	−9.916	0	−4.968	0	−3.768			
	.3090	2.47	.524	.1164	.0610	0	−3.95	9.19	−4.10	0	−.60	0	−.30	0	−.23	.95	1.52	.517
						−12.384	0	−62.917	160.761	−72.472	0	−10.926	0	−5.812	0			
	.4540	2.13	.517	.1040	.0538	−.67	0	−3.38	8.65	−3.90	0	−.59	0	−.31	0	.74	1.39	.517
						0	−8.320	0	−65.803	177.054	−82.083	0	−13.134	0	−7.713			
	.5878	1.73	.500	.0925	.0463	0	−.39	0	−3.05	8.20	−3.80	0	−.61	0	−.36	.60	1.13	.500
						−4.051	0	−7.372	0	−71.743	202.571	−97.965	0	−17.388	0			
	.7071	1.28	.478	.0823	.0393	−.16	0	−.29	0	−2.82	7.96	−3.85	0	−.68	0	.58	.70	.480
						0	−2.880	0	−7.208	0	−81.434	243.694	−125.537	0	−26.635			
	.8090	.80	.441	.0735	.0324	0	−.09	0	−.23	0	−2.64	7.90	−4.07	0	−.86	.61	.19	.443
						−1.638	0	−2.371	0	−7.370	0	−96.962	315.512	−180.528	0			
	.8910	.32	.382	.0665	.0254	−.04	0	−.06	0	−.19	0	−2.46	8.01	−4.59	0	.70	−.38	.386
						0	−1.062	0	−2.016	0	−7.599	0	−122.880	463.533	−329.976			
	.9511	−.10	.292	.0613	.0179	0	−.02	0	−.04	0	−.14	0	−2.20	8.30	−5.91	.89	−.99	.312
						−.459	0	−.620	0	−1.491	0	−7.089	0	−167.045	915.651			
	.9877	−.39	.219	.0437	.0096	0	0	−.01	0	−.01	0	−.07	0	−1.60	8.79	1.33	−1.72	.228
					Σ	1.61	1.07	.95	.74	.60	.58	.61	.70	.89	1.33			

Third approximation						143.239	−58.533	0	−6.950	0	−2.865	0	−1.804	0	−1.468			
	0	3.00	0.497	0.1429	0.0710	10.17	−4.16	0	−.40	0	−.20	0	.13	0	−.10	1.55	1.45	.497
						−115.624	145.025	−67.298	0	−10.158	0	−4.840	0	−3.394	0			
	.1564	2.76	.517	.1295	.0670	−7.75	9.72	−4.51	0	−.68	0	−.32	0	−.23	0	1.12	1.04	.518
						0	−64.802	150.611	−67.157	0	−9.916	0	−4.968	0	−3.768			
	.3090	2.47	.522	.1104	.0608	0	−3.94	9.16	−4.08	0	−.60	0	−.30	0	−.23	.91	1.56	.521
						−12.384	0	−62.917	160.761	−72.472	0	−10.926	0	−5.812	0			
	.4540	2.13	.516	.1040	.0537	−.67	0	−3.38	8.63	−3.89	0	−.59	0	−.31	0	.74	1.39	.517
						0	−8.320	0	−65.803	177.054	−82.083	0	−13.134	0	−7.713			
	.5878	1.73	.500	.0925	.0463	0	−.39	0	−3.05	8.20	−3.80	0	−.61	0	−.36	.60	1.13	.500
						−4.051	0	−7.372	0	−71.743	202.571	−97.965	0	−17.388	0			
	.7071	1.28	.479	.0823	.0394	−.16	0	−.29	0	−2.83	7.98	−3.86	0	−.69	0	.59	.69	.479
						0	−2.880	0	−7.208	0	−81.434	243.694	−125.537	0	−26.635			
	.8090	.80	.443	.0735	.0326	0	−.09	0	−.23	0	−2.65	7.04	−4.09	0	−.87	.62	.18	.442
						−1.638	0	−2.371	0	−7.370	0	−96.962	315.512	−180.528	0			
	.8910	.32	.385	.0665	.0256	−.04	0	−.06	0	−.19	0	−2.48	8.08	−4.62	0	.70	−.38	.386
						0	−1.062	0	−2.016	0	−7.599	0	−122.880	463.533	−329.976			
	.9511	−.10	.299	.0613	.0183	0	−.02	0	−.04	0	−.14	0	−2.25	8.48	−6.04	.99	−1.09	.300
						−.459	0	−.620	0	−1.491	0	−7.089	0	−167.045	915.651			
	.9877	−.39	.224	.0437	.0098	0	0	−.01	0	−.01	0	−.07	0	−1.64	8.97	1.87	−1.76	.224
					Σ	1.55	1.12	.91	.74	.60	.59	.62	.70	.99	1.87			

TABLE VI.—CALCULATION OF WING COEFFICIENTS FOR EXAMPLE WING

[$A=10.05$; $\alpha_s=3.00$]

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫
$\frac{2y}{b}$	Multipliers η_{ms}	$\frac{c_l c}{b}$ (Table V)	α_i (deg) (Table V)	$\frac{57.3 c_{d_i} c}{b}$ (③×④)	c_l (Table V)	c_{d_0} (Section data)	$\frac{c}{\bar{c}}$ (Table IV)	$\frac{c_{d_0} c}{\bar{c}}$ (⑦×⑧)	c_m (Section data)	$\frac{c^2}{\bar{c}c'}$ (Table IV)	$c_m \frac{c^2}{\bar{c}c'}$ (⑩×⑪)
0	.07854	0.0710	1.55	0.1101	0.497	0.0077	1.435	0.0110	−0.081	1.932	−0.156
.1564	.15515	.0670	1.12	.0750	.517	.0078	1.300	.0101	−.081	1.586	−.128
.3090	.14939	.0608	.91	.0553	.522	.0076	1.169	.0089	−.081	1.282	−.104
.4540	.13996	.0537	.74	.0397	.516	.0076	1.041	.0079	−.082	1.022	−.084
.5878	.12708	.0463	.60	.0278	.500	.0076	.929	.0071	−.085	.809	−.069
.7071	.11107	.0394	.59	.0232	.479	.0076	.826	.0063	−.090	.640	−.058
.8090	.09233	.0326	.62	.0202	.443	.0076	.739	.0056	−.092	.512	−.047
.8910	.07131	.0256	.70	.0179	.385	.0076	.668	.0051	−.092	.418	−.038
.9511	.04854	.0183	.99	.0181	.299	.0076	.616	.0047	−.092	.356	−.033
.9877	.02457	.0098	1.37	.0134	.224	.0079	.439	.0035	−.091	.181	−.016

$C_L = A\Sigma(② \times ③) = 0.490$

$C_{D_0} = \Sigma(② \times ⑨) = 0.0077$

$C_{D_i} = \frac{A\Sigma(② \times ⑤)}{57.3} = 0.0078$

$C_m = \Sigma(② \times ⑫) = -0.084$

TABLE VII.—CALCULATION OF LIFT DISTRIBUTION FOR EXAMPLE WING FOR $\epsilon'=0$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
				Multipliers λ_{mk}														
	$\frac{2y}{b}$	α_{a_s}	$\frac{c_l c}{b}$ (Assumed)	k 10	9	8	7	6	5	4	3	2	1	α_i (Σ((3)×(4)) to Σ((3)×(13))) [1]	α_e ((2)−(14))	c_l (a_0×(15))	$\frac{a_0 c}{b}$ (Table IV)	Check $\frac{c_l c}{b}$ ((16)×(17))
				$\frac{2y}{b}$ 0	.1564	.3090	.4540	.5878	.7071	.8090	.8910	.9511	.9877					
First approximation	0	10.000	0.1107	143.239	−58.533	0	−6.950	0	−2.865	0	−1.804	0	−1.468	2.202	7.798	0.7556	0.01385	0.1080
	.1564	↓	.1060	−115.624	145.025	−67.298	0	−10.158	0	−4.840	0	−3.394	0	1.475	8.525	.8295	.01260	.1074
	.3090		.0982	0	−64.802	150.611	−67.157	0	−9.916	0	−4.968	0	−3.768	1.356	8.644	.8454	.01138	.0984
	.4540		.0904	−12.384	0	−62.917	160.761	−72.472	0	−10.926	0	−5.812	0	1.236	8.764	.8624	.01023	.0897
	.5878		.0819	0	−8.320	0	−65.803	177.054	−82.083	0	−13.134	0	−7.713	1.226	8.774	.8695	.00917	.0805
	.7071		.0728	−4.051	0	−7.372	0	−71.743	202.571	−97.965	0	−17.388	0	1.257	8.743	.8734	.00822	.0719
	.8090		.0632	0	−2.880	0	−7.208	0	−81.434	243.694	−125.537	0	−26.635	1.411	8.589	.8649	.00740	.0636
	.8910		.0533	−1.638	0	−2.371	0	−7.370	0	−96.962	315.512	−180.528	0	1.787	8.213	.8328	.00674	.0554
	.9511		.0434	0	−1.062	0	−2.016	0	−7.599	0	−122.880	463.533	−329.976	3.754	6.246	.6371	.00625	.0390
	.9877	↓	.0275	−.459	0	−.620	0	−1.491	0	−7.089	0	−167.045	915.651	8.012	1.988	.2030	.00446	.0089
Second approximation	0	10.000	0.1103	143.239	−58.533	0	−6.950	0	−2.865	0	−1.804	0	−1.468	2.094	7.906	0.7661	0.01385	0.1095
	.1564	↓	.1055	−115.624	145.025	−67.298	0	−10.158	0	−4.840	0	−3.394	0	1.568	8.442	.8214	.01260	.1064
	.3090		.0985	0	−64.802	150.611	−67.157	0	−9.916	0	−4.968	0	−3.768	1.392	8.608	.8419	.01138	.0980
	.4540		.0901	−12.384	0	−62.917	160.761	−72.472	0	−10.926	0	−5.812	0	1.213	8.787	.8646	.01023	.0899
	.5878		.0813	0	−8.320	0	−65.803	177.054	−82.083	0	−13.134	0	−7.713	1.177	8.823	.8744	.00917	.0809
	.7071		.0723	−4.051	0	−7.372	0	−71.743	202.571	−97.965	0	−17.388	0	1.205	8.795	.8786	.00822	.0723
	.8090		.0634	0	−2.880	0	−7.208	0	−81.434	243.694	−125.537	0	−26.635	1.520	8.480	.8539	.00740	.0628
	.8910		.0535	−1.638	0	−2.371	0	−7.370	0	−96.962	315.512	−180.528	0	2.114	7.886	.7996	.00674	.0532
	.9511		.0411	0	−1.062	0	−2.016	0	−7.599	0	−122.880	463.533	−329.976	3.379	6.621	.6753	.00625	.0414
	.9877	↓	.0232	−.459	0	−.620	0	−1.491	0	−7.089	0	−167.045	915.651	4.832	5.168	.5277	.00446	.0230
Third approximation	0	10.000	0.1102	143.239	−58.533	0	−6.950	0	−2.865	0	−1.804	0	−1.468	2.060	7.940	0.7694	0.01385	0.1100
	.1564	↓	.1057	−115.624	145.025	−67.298	0	−10.158	0	−4.840	0	−3.394	0	1.602	8.398	.8171	.01260	.1058
	.3090		.0984	0	−64.802	150.611	−67.157	0	−9.916	0	−4.968	0	−3.768	1.377	8.623	.8433	.01138	.0981
	.4540		.0899	−12.384	0	−62.917	160.761	−72.472	0	−10.926	0	−5.812	0	1.203	8.795	.8654	.01023	.0900
	.5878		.0811	0	−8.320	0	−65.803	177.054	−82.083	0	−13.134	0	−7.713	1.162	8.838	.8758	.00917	.0810
	.7071		.0722	−4.051	0	−7.372	0	−71.743	202.571	−97.965	0	−17.388	0	1.218	8.782	.8773	.00822	.0722
	.8090		.0632	0	−2.880	0	−7.208	0	−81.434	243.694	−125.537	0	−26.635	1.492	8.508	.8568	.00740	.0630
	.8910		.0534	−1.638	0	−2.371	0	−7.370	0	−96.962	315.512	−180.528	0	2.111	7.889	.7999	.00674	.0532
	.9511		.0411	0	−1.062	0	−2.016	0	−7.599	0	−122.880	463.533	−329.976	3.399	6.601	.6733	.00625	.0413
	.9877	↓	.0232	−.459	0	−.620	0	−1.491	0	−7.089	0	−167.045	915.651	4.840	5.160	.5268	.00446	.0230

First assumed $\frac{c_l c}{b} = \frac{\frac{c}{c} + 1.273\sqrt{1-\left(\frac{2y}{b}\right)^2}}{2A+8.6}\, a_0\alpha_{a_s}$.

[1] $\alpha_i = \Sigma(3)\times\lambda_{mk}$.

TABLE VIII.—CALCULATION OF LIFT DISTRIBUTION FOR EXAMPLE WING FOR $\alpha_{a_s}=0$

	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭	⑮	⑯	⑰	⑱
				Multipliers λ_{mk}														
	$\frac{2y}{b}$	ϵ' (Table IV)	$\frac{c_l c}{b}$ (Assumed)	k 10	9	8	7	6	5	4	3	2	1	α_i (Σ(③×④) to (Σ③×⑬))	α_s (②−⑭)	c_l (α_0×⑮)	$\frac{a_0 c}{b}$ (Table IV)	Check $\frac{c_l c}{b}$ (⑯×⑰)
				$\frac{2y}{b}$ 0	.1564	.3090	.4540	.5878	.7071	.8090	.8910	.9511	.9877	(1)				
First approximation	0	0	0	143.239	−58.533	0	−6.950	0	−2.865	0	−1.804	0	−1.468	0.460	−0.460	−0.0446	0.01385	−0.0064
	.1564	−.235	−.0025	−115.62	145.025	−67.298	0	−10.158	0	−4.840	0	−3.394	0	.105	−.340	−.0331	.01260	−.0043
	.3090	−.516	−.0051	0	−64.802	150.611	−67.157	0	−9.916	0	−4.968	0	−3.768	.012	−.528	−.0516	.01138	−.0060
	.4540	−.849	−.0077	−12.384	0	−62.917	160.761	−72.472	0	−10.926	0	−5.812	0	−.107	−.742	−.0730	.01023	−.0076
	.5878	−1.235	−.0101	0	−8.320	0	−65.803	177.054	−82.083	0	−13.134	0	−7.713	−.221	−1.014	−.1005	.00917	−.0098
	.7071	−1.670	−.0121	−4.051	0	−7.372	0	−71.743	202.571	−97.965	0	−17.388	0	−.373	−1.297	−.1296	.00822	−.0107
	.8090	−2.138	−.0135	0	−2.880	0	−7.208	0	−81.434	243.694	−125.537	0	−26.635	−.596	−1.542	−.1553	.00740	−.0114
	.8910	−2.604	−.0139	−1.638	0	−2.371	0	−7.370	0	−96.962	315.512	−180.528	0	−.923	−1.681	−.1705	.00674	−.0113
	.9511	−3.013	−.0131	0	−1.062	0	−2.016	0	−7.599	0	−122.880	463.533	−329.976	−1.779	−1.234	−.1259	.00625	−.0077
	.9877	−3.297	−.0091	−.459	0	−.620	0	−1.491	0	−7.089	0	−167.045	915.651	−3.553	.256	−.0261	.00446	.0011
Second approximation	0	0	−0.0023	143.239	−58.533	0	−6.950	0	−2.865	0	−1.804	0	−1.468	0.275	−0.275	−0.0266	0.01385	−0.0038
	.1564	−.235	−.0038	−115.624	145.025	−67.298	0	−10.158	0	−4.840	0	−3.394	0	.075	−.310	−.0302	.01260	−.0039
	.3090	−.516	−.0056	0	−64.802	150.611	−67.157	0	−9.916	0	−4.968	0	−3.768	.014	−.530	−.0518	.01138	−.0060
	.4540	−.849	−.0077	−12.384	0	−62.917	160.761	−72.472	0	−10.926	0	−5.812	0	−.095	−.754	−.0742	.01023	−.0077
	.5878	−1.235	−.0097	0	−8.320	0	−65.803	177.054	−82.083	0	−13.134	0	−7.713	−.202	−1.033	−.1024	.00917	−.0095
	.7071	−1.670	−.0114	−4.051	0	−7.372	0	−71.743	202.571	−97.965	0	−17.388	0	−.350	−1.320	−.1319	.00822	−.0109
	.8090	−2.138	−.0125	0	−2.880	0	−7.208	0	−81.434	243.694	−125.537	0	−26.635	−.571	−1.567	−.1578	.00740	−.0116
	.8910	−2.604	−.0124	−1.638	0	−2.371	0	−7.370	0	−96.962	315.512	−180.528	0	−.844	−1.760	−.1785	.00674	−.0119
	.9511	−3.013	−.0109	0	−1.062	0	−2.016	0	−7.599	0	−122.880	463.533	−329.976	−1.456	−1.557	−.1588	.00625	−.0097
	.9877	−3.297	−.0066	−.459	0	−.620	0	−1.491	0	−7.089	0	−167.045	915.651	−2.014	−1.283	−.1310	.00446	−.0057
Third approximation	0	0	−0.0029	143.239	−58.533	0	−6.950	0	−2.865	0	−1.804	0	−1.468	0.210	−0.210	−0.0203	0.01385	−0.0029
	.1564	−.235	−.0040	−115.624	145.025	−67.298	0	−10.158	0	−4.840	0	−3.394	0	.085	−.320	−.0311	.01260	−.0040
	.3090	−.516	−.0057	0	−64.802	150.611	−67.157	0	−9.916	0	−4.968	0	−3.768	.009	−.525	−.0518	.01138	−.0060
	.4540	−.849	−.0077	−12.384	0	−62.917	160.761	−72.472	0	−10.926	0	−5.812	0	−.095	−.754	−.0743	.01023	−.0077
	.5878	−1.235	−.0096	0	−8.320	0	−65.803	177.054	−82.083	0	−13.134	0	−7.713	−.207	−1.028	−.1019	.00917	−.0094
	.7071	−1.670	−.0111	−4.051	0	−7.372	0	−71.743	202.571	−97.965	0	−17.388	0	−.331	−1.339	−.1338	.00822	−.0110
	.8090	−2.138	−.0121	0	−2.880	0	−7.208	0	−81.434	243.694	−125.537	0	−26.635	−.550	−1.588	−.1599	.00740	−.0118
	.8910	−2.604	−.0120	−1.638	0	−2.371	0	−7.370	0	−96.962	315.512	−180.528	0	−.830	−1.774	−.1799	.00674	−.0120
	.9511	−3.013	−.0104	0	−1.062	0	−2.016	0	−7.599	0	−122.880	463.533	−329.976	−1.351	−1.662	−.1695	.00625	−.0104
	.9877	−3.297	−.0063	−.459	0	−.620	0	−1.491	0	−7.089	0	−167.045	915.651	−1.915	−1.382	−.1411	.00446	−.0062

First assumed $\frac{c_l c}{b} = \frac{\frac{c}{\bar{c}} + 1.273\sqrt{1-\left(\frac{2y}{b}\right)^2}}{2A+3.6}a_0\epsilon'$.

[1] $\alpha_{i_k} = \Sigma ③ \times \lambda_{mk}$.

TABLE IX.—CALCULATION OF LINEAR LIFT CHARACTERISTICS FOR EXAMPLE WING

[$A=10.05$; $\alpha_{a_s}=10.00$; $\alpha_{l_{0_s}}=-3.90$]

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫
$\frac{2y}{b}$	Multipliers η_{ms}	$\left(\frac{c_l c}{b}\right)_{(\alpha_{a_s})}$ (Table VII)	$\frac{c_{l_{a1}} c}{b}$ $\left(\frac{③}{C_{L_{(\alpha_{a_s})}}}\right)$	$\left(\frac{c_l c}{b}\right)_{(\epsilon_t')}$ (Table VIII)	$\frac{c_{l_{a1}} c}{b} C_{L_{(\epsilon_t')}}$ (④$\times C_{L_{(\epsilon_t')}}$)	$\frac{c_{l_b} c}{b}$ (⑤−⑥)	$\frac{c}{b}$ (Table IV)	$c_{l_{a1}}$ $\left(\frac{④}{⑧}\right)$	c_{l_b} $\left(\frac{⑦}{⑧}\right)$	$c_{l_{max}}$ (Section data)	$\frac{c_{l_{max}}-c_{l_b}}{c_{l_{a1}}}$ $\left(\frac{⑪-⑩}{⑨}\right)$
0	0.07854	0.1102	0.1323	−0.0029	−0.0105	0.0076	0.1429	0.926	0.053	1.421	1.477
.1564	.15515	.1057	.1269	−.0040	−.0100	.0060	.1295	.980	.046	1.418	1.400
.3090	.14939	.0984	.1181	−.0057	−.0093	.0036	.1164	1.015	.031	1.423	1.371
.4540	.13996	.0899	.1079	−.0077	−.0085	.0008	.1040	1.038	.008	1.432	1.372
.5878	.12708	.0811	.0974	−.0096	−.0077	−.0019	.0925	1.053	−.021	1.441	1.388
.7071	.11107	.0722	.0867	−.0111	−.0068	−.0043	.0823	1.053	−.051	1.436	1.412
.8090	.09233	.0632	.0759	−.0121	−.0060	−.0061	.0735	1.033	−.083	1.418	1.453
.8910	.07131	.0534	.0641	−.0120	−.0051	−.0069	.0665	.964	−.104	1.404	1.564
.9511	.04854	.0411	.0493	−.0104	−.0039	−.0065	.0613	.804	−.106	1.419	1.897
.9877	.02457	.0232	.0279	−.0063	−.0022	−.0041	.0437	.633	−.094	1.412	2.361

$C_{L_{(\alpha_{a_s})}}=A\Sigma(②\times③)=0.833$

$a=\frac{C_{L_{(\alpha_{a_s})}}}{\alpha_{a_s}}=0.0833$

$C_{L_{max}}=$Min. value in ⑫$=1.37$

$C_{L_{(\epsilon_t')}}=A\Sigma(②\times⑤)=-0.079$

$\alpha_{a_{s(L=0)}}=\frac{-C_{L_{(\epsilon_t')}}}{a}=0.95$

$\alpha_{s(L=0)}=\alpha_{l_{0_s}}+\alpha_{a_{s(L=0)}}=-2.95$

TABLE X.—CALCULATION OF INDUCED-DRAG COEFFICIENT FOR EXAMPLE WING

[$A=10.05$; $C_{L_{(\alpha_{a_s})}}=0.833$; $C_{L_{(\epsilon_t')}}=-0.079$]

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫
$\frac{2y}{b}$	Multipliers η_{ms}	$\alpha_{i_{(\alpha_{a_s})}}$ (Table VII)	$\alpha_{i_{a1}}$ $\left(\frac{③}{C_{L_{(\alpha_{a_s})}}}\right)$	$\alpha_{i_{(\epsilon_t')}}$ (Table VIII)	$\alpha_{i_{a1}} C_{L_{(\epsilon_t')}}$ (④$\times C_{L_{(\epsilon_t')}}$)	α_{i_b} (⑤−⑥)	$\frac{c_{l_{a1}} c}{b}$ (Table IX)	$\frac{c_{l_b} c}{b}$ (Table IX)	$\frac{57.3 c_{d_{i_{a1}}} c}{b}$ (④×⑧)	$\frac{57.3 c_{d_{i_{a1b}}} c}{b}$ ((④×⑨)+(⑦×⑧))	$\frac{57.3 c_{d_{i_b}}}{b}$ (⑦×⑨)
0	0.07854	2.060	2.474	0.210	−0.195	0.405	0.1323	0.0076	0.3273	0.0724	0.0031
.1564	.15515	1.602	1.924	.085	−.152	.237	.1269	.0060	.2142	.0416	.0014
.3090	.14939	1.377	1.653	.009	−.131	.140	.1181	.0036	.1952	.0225	.0005
.4540	.13996	1.203	1.445	−.095	−.114	.019	.1079	.0008	.1559	.0032	0
.5878	.12708	1.162	1.395	−.207	−.110	−.097	.0974	−.0019	.1359	−.0121	.0002
.7071	.11107	1.218	1.463	−.331	−.116	−.215	.0867	−.0042	.1268	−.0248	.0009
.8090	.09233	1.492	1.792	−.550	−.142	−.408	.0759	−.0061	.1360	−.0419	.0025
.8910	.07131	2.111	2.535	−.830	−.200	−.630	.0641	−.0069	.1625	−.0579	.0043
.9511	.04854	3.399	4.081	−1.351	−.322	−1.029	.0493	−.0065	.2012	−.0773	.0067
.9877	.02457	4.840	5.812	−1.915	−.459	−1.456	.0279	−.0041	.1622	−.0615	.0060

$C_{D_i}=\left(\frac{A\Sigma(②\times⑩)}{57.3}\right)C_L^2+\left(\frac{A\Sigma(②\times⑪)}{57.3}\right)C_L+\frac{A\Sigma(②\times⑫)}{57.3}$

$=0.0322C_L^2-0.0003C_L+0.0003$

The values of $\alpha_{i_{a1}}$ and α_{i_b} are determined in table X in the same manner as $\frac{c_{l_{a1}}c}{b}$ and $\frac{c_{l_b}c}{b}$ in table IX. The induced-drag distribution is therefore

$$\frac{c_{d_i}c}{b}=\frac{c_l c}{b}\frac{\alpha_i}{57.3}$$

or

$$\frac{c_{d_i}c}{b}=\frac{c_{d_{i_{a1}}}c}{b}C_L^2+\frac{c_{d_{i_{a1b}}}c}{b}C_L+\frac{c_{d_{i_b}}c}{b} \tag{25}$$

where

$$\frac{c_{d_{i_{a1}}}c}{b}=\frac{c_{l_{a1}}c}{b}\frac{\alpha_{i_{a1}}}{57.3} \tag{26}$$

$$\frac{c_{d_{i_{a1b}}}c}{b}=\frac{c_{l_{a1}}c}{b}\frac{\alpha_{i_b}}{57.3}+\frac{c_{l_b}c}{b}\frac{\alpha_{i_{a1}}}{57.3} \tag{27}$$

and

$$\frac{c_{d_{i_b}}c}{b}=\frac{c_{l_b}c}{b}\frac{\alpha_{i_b}}{57.3} \tag{28}$$

The calculation of each of these induced-drag distributions is illustrated in table X together with the numerical integration of each distribution to obtain the wing induced-drag coefficient.

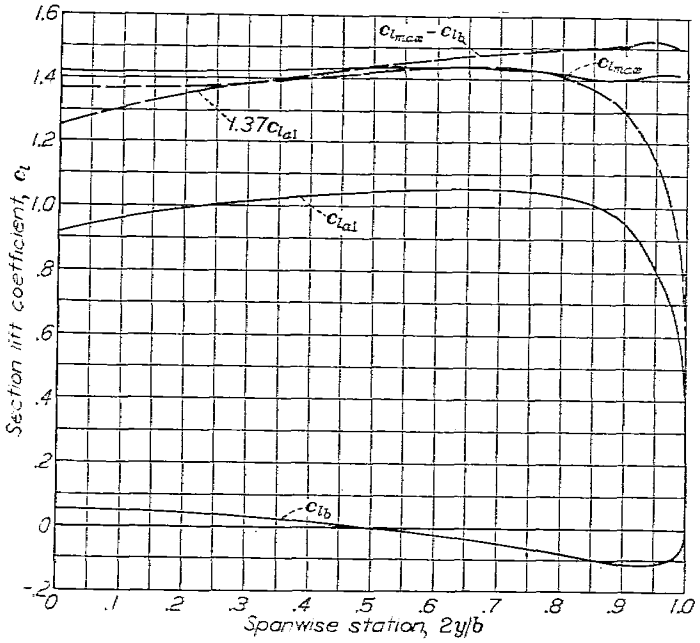


FIGURE 4.—Estimation of $C_{L_{max}}$ for example wing. ($C_{L_{max}}$ estimated to be 1.37.)

PROFILE-DRAG AND PITCHING-MOMENT COEFFICIENTS

The profile-drag and pitching-moment coefficients for the wing depend directly upon the section data and therefore their calculation is the same whether linear or nonlinear section lift data are used. For the linear case the section lift coefficient is

$$c_l = c_{l_{a1}} C_L + c_{l_b}$$

for any wing coefficient C_L. By use of this value for c_l the profile-drag and pitching-moment coefficients are found as in table VI.

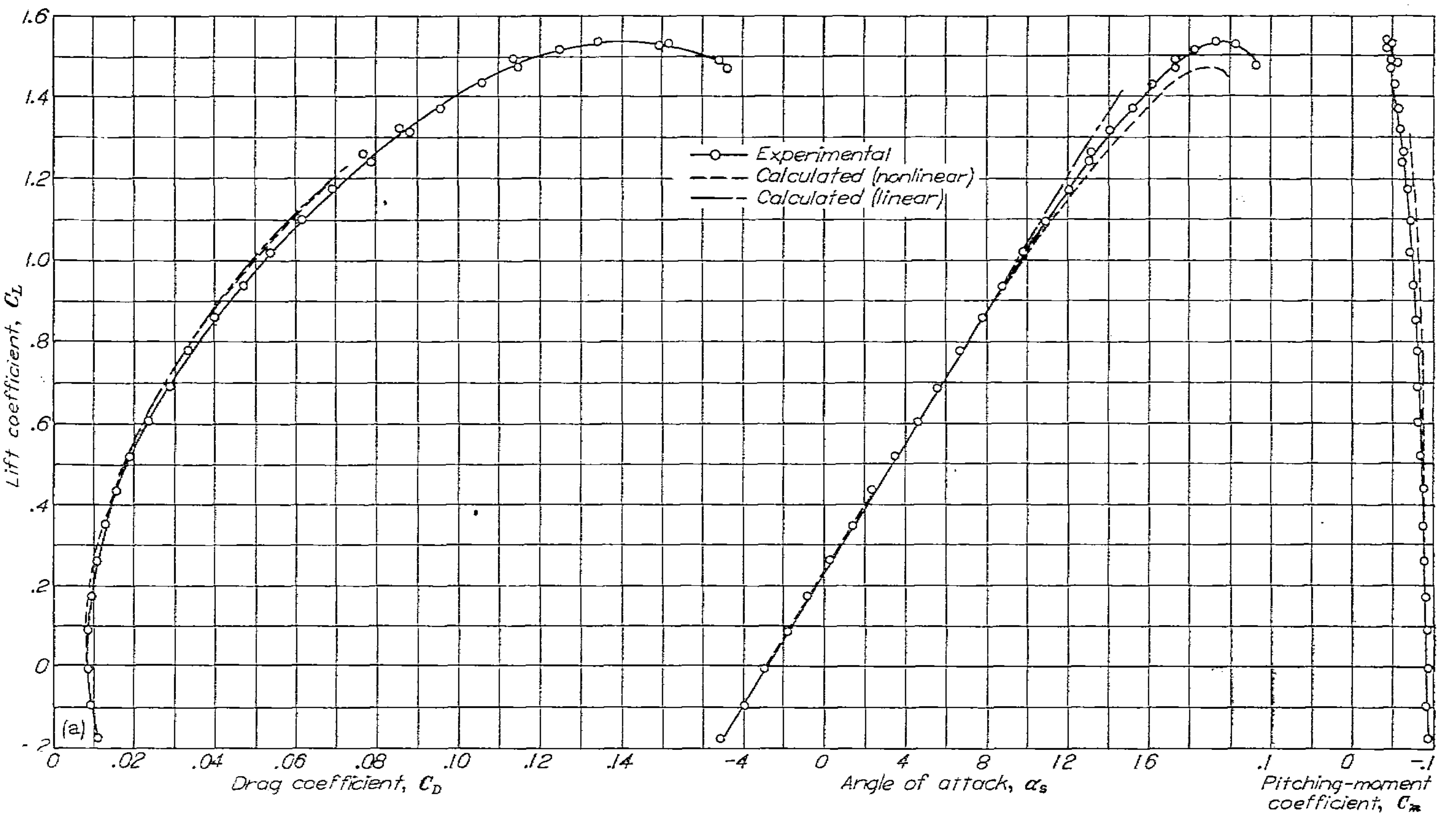
DISCUSSION

The characteristics of three wings with symmetrical lift distributions have been calculated by use of both nonlinear and linear section lift data and are presented in figure 5 together with experimental results. These data were taken from reference 11. The lift curves calculated by use of nonlinear section lift data are in close agreement with the experimental results over the entire range of lift coefficient, whereas those calculated by use of linear section lift data are in agreement only over the linear portions of the curves as would be expected.

It must be remembered that the methods presented are subject to the limitations of lifting-line theory upon which the methods are based; therefore, the close agreement shown in figure 5 should not be expected for wings of low aspect ratio or large sweep. The use of the edge-velocity factor more or less compensates for some of the effects of aspect ratio and, in fact, appears to overcompensate at the larger values of aspect ratio as shown in figure 5.

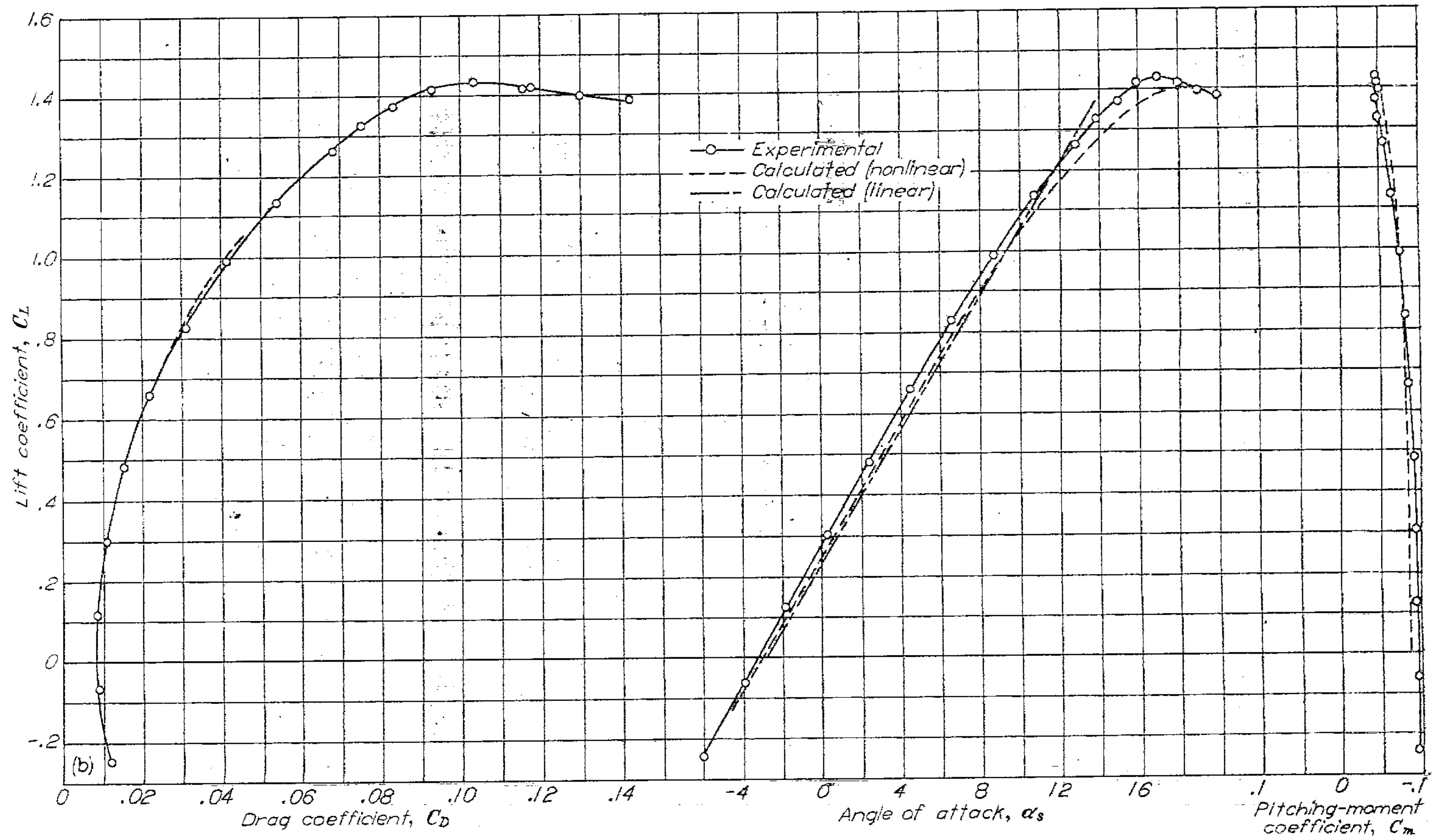
Additional comparisons of calculated and experimental data are given in reference 11 for wings with symmetrical lift distributions, but very little comparable data are available for wings with asymmetrical lift distributions. Such data are very desirable in order to determine the reliability with which calculated data may be used to predict experimental wing characteristics.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *December 20, 1946.*



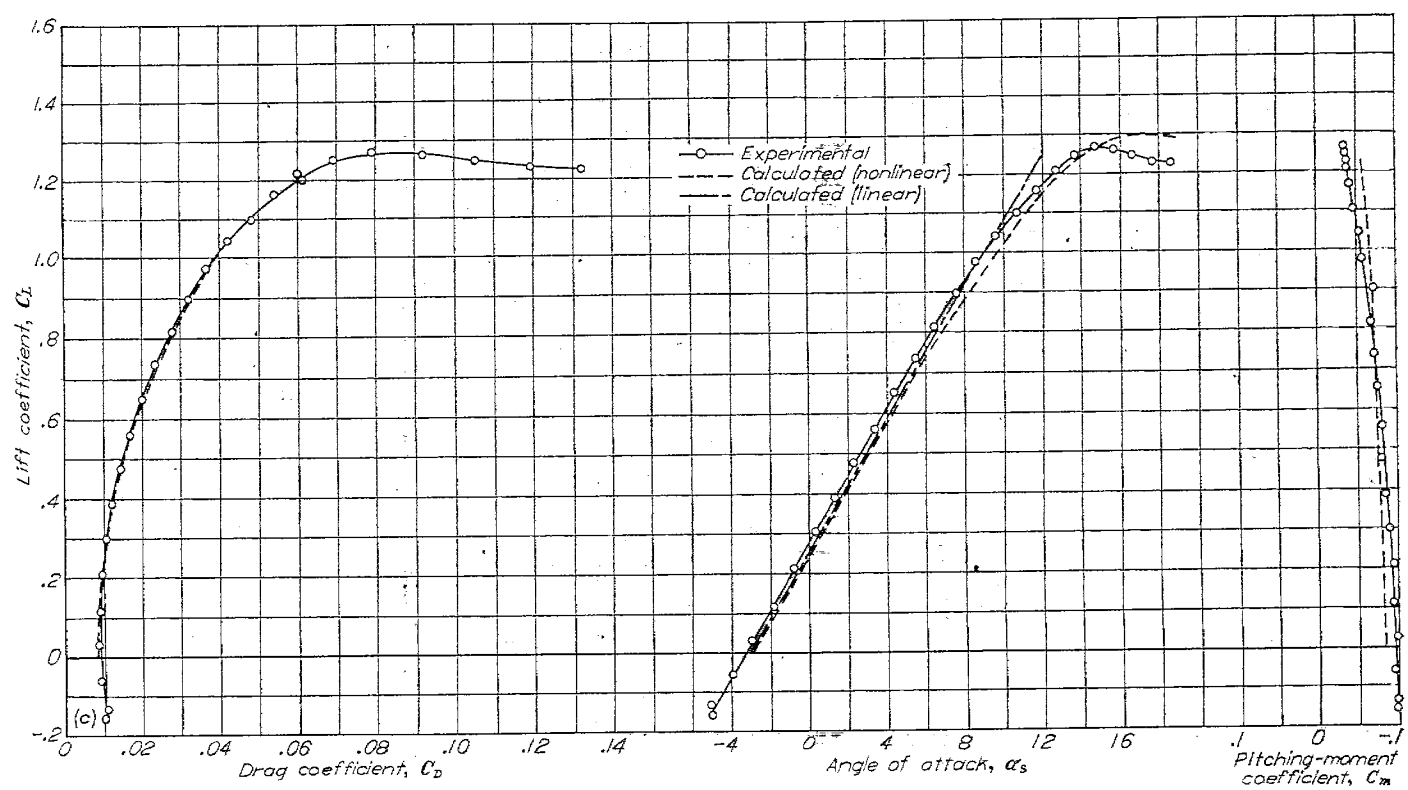
(a) $A=8.04$; $R=4.32\times10^6$; root section, NACA 4416; tip section, NACA 4412.

FIGURE 5.—Experimental and calculated characteristics of three wings of taper ratio 2.5 and NACA 44-series airfoil sections.



(b) $A=10.05$; $R=3.49\times10^6$; root section, NACA 4420; tip section, NACA 4412.

FIGURE 5.—Continued.



(c) $A=12.06$; $R=2.87\times10^6$; root section, NACA 4424; tip section, NACA 4412.

FIGURE 5.—Concluded.

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